INFLUENCE OF A MAGNETIC FIELD ON A NONISOTHERMAL LUBRICATING FILM OF A CONDUCTING GAS

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The formulation of the problem of nonisothermal gas lubrication on the basis of the Reynolds equations is given in [1,2], together with some solutions to this problem. Of practical and theoretical interest is the case in which external magnetic and electric fields are present and the lubricating medium is electrically conducting. A rigorous formulation of this problem for an incompressible conducting fluid has been given by Shukla [3], and for an isothermal gas film by Constantinesku [4], who also obtained a number of quantitative estimates.

In the present paper, the problem of three-dimensional nonisothermal electrically conducting gas lubrication in the presence of a transverse magnetic field is formulated, and basic characteristics of the problem are examined.

Assume we have a gas film, a portion of which is schematically shown in Fig. 1. The surface situated in the plane x0z moves at a constant velocity U in direction of the x-axis, while the surface opposite to it is at rest. Neither surface is electrically conducting. An electrical conductivity σ of the lubricating gas is provided for by some means (influence of radiation, introduction of ionizing additions, etc.); we assume that σ = const.

The entire system is situated in an external magnetic field H_0 which is oriented along the y-axis. Only cases with small magnetic Reynolds numbers R_m where the induced magnetic field may be neglected (considering $H \equiv H_0$ = const) will be treated.

We introduce dimensionless designations for the gas velocity components u, v, and w, the density ρ, the temperature T, and the pressure p. The values at the surface xz and the pressure p_0 at a certain line parallel to the y-axis are selected as the scales. The length scale in the x-, and z-directions is the characteristic dimension of the gas bearing l, and the scale in the y-direction is the mean thickness of the clearance h_0.

In addition, we introduce the generalized dimensionless quantities

$$P = \frac{\mu c_p}{\lambda}, \quad M = \frac{U}{\sqrt{RT_0}},$$
$$\Lambda = \frac{U\mu_0 l}{h_c^2 p_0}, \quad G = h_0 \mu_e H_0 \sqrt{\frac{\sigma}{\mu_0}},$$

where P is the Prandtl number, M is the Mach number, Λ is the characteristic value of the gas bearing, and G is the Hartmann number.

In [1] it was shown that if the value of $\alpha = (\varkappa - 1)\,\mathrm{Pr}\, M^2$ is small and the viscosity coefficient and temperature are governed by a power-law relation, then, in the approximation of gas lubrication theory (the hydrodynamic Reynolds number Re = 0 (1), $(h_0/l)^2 \ll 1$), the energy equations lead to an expression for the temperature

$$T = \left[(\chi^{n+1} - 1)\frac{y}{h} + 1\right]^{\frac{1}{n+1}}, \tag{1}$$

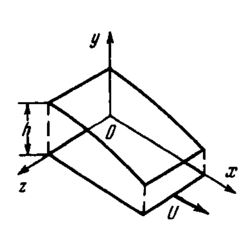

Fig. 1.

where χ is the constant temperature ratio of the moving and fixed surfaces, and n is the exponent in the power-law relation between viscosity and temperature. Turning to the magnetogasdynamic problem, it is not difficult to see that the ratio of the term which characterizes Joule dissipation to the principal term associated with thermal conductivity in the energy equation is proportional to the quantity

$$\beta = (\varkappa - 1)\, P M^2 G^2.$$

Consequently, when the parameter β is small, Joule dissipation does not affect the temperature, and expression (1) continues to hold. The following considerations will deal specifically with this case.

Having thus eliminated the need for analyzing the energy equation, the analysis may be limited to the equation of state, three projections of the equations of motion, and the continuity equation. By introducing the additional assumption that the lubricating medium is a perfect gas, we get

$$p = \frac{\varkappa - 1}{\varkappa}\rho T, \qquad \frac{\partial}{\partial y}\left(T^n \frac{\partial u}{\partial y}\right) - G^2 u = \frac{1}{\Lambda}\frac{\partial p}{\partial x},$$
$$\frac{\partial p}{\partial y} = 0, \qquad \frac{\partial}{\partial y}\left(T^n \frac{\partial w}{\partial y}\right) - G^2 w = \frac{1}{\Lambda}\frac{\partial p}{\partial z},$$
$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0. \tag{2}$$

As already stated, the case T = const was studied in [4,5] and, therefore, will not be considered. In the case of a variable temperature within the lubrication film, i.e., for $\chi \neq 1$, Eq. (1) yields a single-valued relationship T = T(y) for a fixed h (fixed values of x and z). Because of this, one may change in the equations of motion from the variable y to a new independent variable T. In this case, we have

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial T}\frac{\partial T}{\partial y} = \frac{\chi^{n+1} - 1}{(n+1)h}\left[(\chi^{n+1} - 1)\frac{y}{h} + 1\right]^{-\frac{n}{n+1}}\frac{\partial}{\partial T} =$$
$$= \frac{\chi^{n+1} - 1}{(n+1)h} T^{-n}\frac{\partial}{\partial T},$$
$$\frac{\partial}{\partial x} = \frac{\partial}{\partial x} + \frac{\partial}{\partial T}\frac{\partial T}{\partial x}, \qquad \frac{\partial}{\partial z} = \frac{\partial}{\partial z} + \frac{\partial}{\partial T}\frac{\partial T}{\partial z}.$$

The quantities $\partial T/\partial x$ and $\partial T/\partial z$ are assumed to be finite. Under this assumption, system (2) is transformed as follows:

$$p = \frac{\varkappa - 1}{\varkappa}\rho T, \quad \frac{\partial^2 u}{\partial T^2} - G^2\frac{h^2(n+1)^2}{(\chi^{n+1} - 1)^2}T^n u =$$
$$= \frac{1}{\Lambda}\frac{\partial p}{\partial x}\frac{h^2(n+1)^2}{(\chi^{n+1} - 1)^2}T^n, \ \frac{\partial p}{\partial T} = 0,$$
$$\frac{\partial^2 w}{\partial T^2} - G^2\frac{h^2(n+1)^2}{(\chi^{n+1} - 1)^2}T^n w = \frac{1}{\Lambda}\frac{\partial p}{\partial z}\frac{h^2(n+1)^2}{(\chi^{n+1} - 1)^2}T^n,$$
$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0. \tag{3}$$

The first step of the solution reduces to the integration of the second and fourth equations in (3), i.e., to the representation of the velocities u and w in the form of expressions which contain the pressure derivatives $\partial p/\partial x$ and $\partial p/\partial z$. The equations for u and w have a similar structure. A solution of one of them is

$$\frac{\partial^2 u}{\partial T^2} - G^2 \frac{h^2(n+1)^2}{(\chi^{n+1}-1)^2} T^n u = \frac{1}{\Lambda} \frac{\partial p}{\partial x} \frac{h^2(n+1)^2}{(\chi^{n+1}-1)^2} T^n. \tag{4}$$

Because the quantities p and h depend only on x and z, the equation lends itself to integration over one independent variable, namely T. The equation is linear and, hence, the general solution is the sum of the general solution without the right-hand side and the particular solution with the right-hand side. It is well known (see, for example, [6]) that the solution of an equation of this type without the right-hand side can be expressed in Bessel functions in the form

$$u^{(0)} = \sqrt{T} Z_\nu \left(\frac{2}{n+2} G \frac{h(n+1)}{(\chi^{n+1}-1)} T^{\frac{n+2}{2}} \right), \qquad \nu = \frac{1}{n+2}. \tag{5}$$

In formula (5) Z_ν denotes a linear combination of Bessel functions of a purely imaginary argument

$$Z_\nu(\xi) = C_1 I_\nu(\xi) + C_2 I_{-\nu}(\xi),$$

$$\xi = 2G \frac{(n+1)h}{(n+2)(\chi^{n+1}-1)} T^{\frac{n+2}{2}}. \tag{6}$$

With the aid of the method of varying the constants, and by using the expression for the Wronskian of a complex of fundamental solutions of the modified Bessel equation [7]

$$I_\nu'(\xi) I_{-\nu}(\xi) - I_\nu(\xi) I_{-\nu}'(\xi) = \frac{2 \sin \nu\pi}{\pi\xi},$$

together with a related expression

$$I_{1-\nu}(\xi) I_\nu(\xi) - I_{\nu-1}(\xi) I_{-\nu}(\xi) = -\frac{2 \sin \nu\pi}{\pi\xi},$$

it is possible to obtain also a particular solution of Eq. (5) with the right-hand side. As a result, the general solution of equation (3) with respect to u and w takes the form

$$u = \sqrt{T}[f_1(x,z) I_\nu(\xi) + f_2(x,z) I_{-\nu}(\xi)] - \frac{1}{G^2\Lambda} \frac{\partial p}{\partial x},$$

$$w = \sqrt{T}[f_3(x,z) I_\nu(\xi) + f_4(x,z) I_{-\nu}(\xi)] - \frac{1}{G^2\Lambda} \frac{\partial p}{\partial z}. \tag{7}$$

The arbitrary functions f_1, f_2, f_3 and f_4 are determined from the boundary conditions. For the temperature and the velocity components, these boundary conditions have the form

$$T = 1, \quad u = 1, \quad v = w = 0 \quad \text{for } y = 0,$$

$$T = \chi, \quad u = v = w = 0 \quad \text{for } y = h. \tag{8}$$

From formula (6), we have

$$\xi = \xi_0 = 2G \frac{(n+1)h}{(n+2)(\chi^{n+1}-1)} \quad \text{for } y = 0,$$

$$\xi = \xi_0 \chi^{1/2(n+2)} \quad \text{for } y = h. \tag{8a}$$

From Eqs. (8) and (8a), we get

$$f_1 = -\frac{1}{A_\nu}\left[I_{-\nu}(\chi^{\frac{1}{2\nu}}\xi_0) + \frac{1}{G^2\Lambda}\frac{\partial p}{\partial x} C_{-\nu} \right],$$

$$f_2 = \frac{1}{A_\nu}\left[I_\nu(\chi^{\frac{1}{2\nu}}\xi_0) + \frac{1}{G^2\Lambda}\frac{\partial p}{\partial x} C_\nu \right],$$

$$f_3 = -\frac{1}{A_\nu G^2 \Lambda}\frac{\partial p}{\partial z}\left[I_{-\nu}(\chi^{\frac{1}{2\nu}}\xi_0) - \chi^{-\frac{1}{2}} I_{-\nu}(\xi_0) \right],$$

$$f_4 = \frac{1}{A_\nu G^2 \Lambda}\frac{\partial p}{\partial z}\left[I_\nu(\chi^{\frac{1}{2\nu}}\xi_0) - \chi^{-\frac{1}{2}} I_\nu(\xi_0) \right],$$

$$A_\nu = I_\nu(\chi^{\frac{1}{2\nu}}\xi_0) I_{-\nu}(\xi_0) - I_{-\nu}(\xi_0) I_{-\nu}(\chi^{\frac{1}{2\nu}}\xi_0),$$

$$C_{\pm\nu} = I_{\pm\nu}(\chi^{\frac{1}{2\nu}}\xi_0) - \chi^{\frac{1}{2}} I_{\pm\nu}(\xi_0). \tag{9}$$

The second step of the solution involves the substitution of expressions (7) into the continuity equation, i.e., the last equation in system (3), which can be more conveniently represented in integral form:

$$\frac{\partial}{\partial x}\left(p \int_0^h \frac{u}{T} dy \right) + \frac{\partial}{\partial z}\left(p \int_0^h \frac{w}{T} dy \right) = 0. \tag{10}$$

Integration of the functions u/T and w/T, obtained from (7), can be simplified by introducing the substitution

$$dy = \frac{h(n+1)}{\chi^{n+1}-1} T^n dT = \frac{1}{G} T^{\frac{n}{2}} d\xi.$$

Transformation of Eq. (10) into an equation for determining the pressure involves introduction of certain integrals of the form

$$D_{\pm\nu}(\xi) = \int_\alpha^\xi \xi^{1-3\nu} I_{\pm\nu}(\xi)\, d\xi, \tag{11}$$

which do not lend themselves to analytical representation. Here, α is a constant which is not equal to any of the values of ξ_0 or $\chi^{1/2\nu}\xi_0$, but which is close to one of these quantities in order of magnitude. Let us also introduce the notation

$$B_{\pm\nu} = D_{\pm\nu}(\chi^{\frac{1}{2\nu}}\xi_0) - D_{\pm\nu}(\xi_0). \tag{12}$$

By using, in addition, the previously introduced quantities A_ν and $C_{\pm\nu}$, the above equation can be written in the form

$$\frac{\partial}{\partial x}\left\{ \left\{ h^{3\nu-1} p \left\{ \frac{1}{G^2\Lambda}\frac{\partial p}{\partial x}\left[\frac{B_{-\nu}C_\nu - B_\nu C_{-\nu}}{A_\nu} - \right.\right.\right.\right.$$

$$\left. - \frac{1}{2(1-2\nu)}\left(\frac{2G(1-\nu)h}{\chi^{(1-\nu)/\nu}-1} \right)^{2-3\nu} (\chi^{\frac{1-2\nu}{\nu}} - 1) \right] +$$

$$\left.\left.\left. + \frac{1}{A_\nu}[B_{-\nu} I_\nu(\chi^{\frac{1}{2\nu}}\xi_0) - B_\nu I_{-\nu}(\chi^{\frac{1}{2\nu}}\xi_0)] \right\}\right\}\right\} +$$

$$+ \frac{\partial}{\partial z}\left\{ h^{3\nu-1} \frac{1}{G^2\Lambda} p \frac{\partial p}{\partial z}\left[\frac{B_{-\nu}C_\nu - B_\nu C_{-\nu}}{A_\nu} - \right.\right.$$

$$\left.\left. - \frac{1}{2(1-2\nu)}\left(\frac{2G(1-\nu)h}{\chi^{(1-\nu)/\nu}-1} \right)^{2-3\nu} (\chi^{\frac{1-2\nu}{\nu}} - 1) \right]\right\} = 0. \tag{13}$$

If the subscript ν is known (or, which is the same, if the exponent n in the power-law relation between viscosity and temperature is known, and the Hartmann number G, the characteristic value Λ of the gas bearing, and the temperature ratio of the surfaces, χ, are all known, then for a certain function h(x, z) which characterizes the shape of the clearance, and for certain boundary conditions, it is possible, from Eq. (13), to determine the pressure p at any line parallel to the y-axis. It can be shown that in the limiting case $G \to 0$, Eq. (13) coincides with an analogous equation in nonisothermal gas lubrication theory [1].

As has been shown [1,2], the solution of Eq. (13), even in the simple case G = 0, is realized by numerical methods or with the aid of various simplifications. This is all the more true for $G \neq 0$.

Of interest is a specific feature of the behavior of the pressure in the case $\Lambda \to \infty$, which derives from Eq. (13). In cases in which G = 0 and $G \neq 0$, but where $\chi = 1$ (see [5]), the passage to the limit $\Lambda \to \infty$ leads to the condition

$$ph = \text{const}. \tag{14}$$

Equation (13) shows, however, that in a nonisothermal gas film with a finite Hartmann number, the asymptotic condition (14) no longer holds. It is replaced by an appreciably more complex condition

$$p h^{3\nu-1} \frac{B_{-\nu} I_\nu(\chi^{\frac{1}{2\nu}}\xi_0) - B_\nu I_{-\nu}(\chi^{\frac{1}{2\nu}}\xi_0)}{A_\nu} = \text{const}. \tag{15}$$

As can be seen from formulas (9), (11), and (12), the quantities A_ν and $B_{\pm\nu}$ contain certain combinations of Bessel functions of a purely imaginary argument and of integrals which incorporate such functions, but which cannot generally be represented in analytical form. Furthermore, the tabular form of the functions $I_{\pm\nu}(\xi)$ themselves is known only for certain discrete values of the exponent n (for example, n = 1, which corresponds to $\nu = 1/3$). One of the simplest ways of solving Eq. (13) in the general case of an arbitrary n, however, involves representation of the Bessel functions $I_{\pm\nu}(\xi)$ in convergent power series [7]

$$I_{\pm\nu}(\xi)=\sum_{r=0}^{\infty}\frac{1}{r!\,\Gamma(1+r\pm\nu)}\left(\frac{\xi}{2}\right)^{2r\pm\nu}. \tag{16}$$

By using series (16), the quantities A_ν and $C_{\pm\nu}$, defined by formulas (9), can be easily calculated for any value of $\nu = (n+2)^{-1}$. By substituting these series into the integrands of formulas (11), we obtain

$$D_{\pm\nu}(\xi)=$$

$$=2^{2-3\nu}\sum_{r=0}^{\infty}\frac{1}{r!\,(2r+2-3\nu\pm\nu)\,\Gamma(1+r\pm\nu)}\left(\frac{\xi}{2}\right)^{2r+2-3\nu\pm\nu}\Bigg|_{x}^{\xi}. \tag{17}$$

Evaluating the right-hand sides of formula (17) with the required accuracy, and using (12), it is not difficult to find $B_{\pm\nu}$ and to obtain all data for solving Eq. (13).

All the considerations we have made refer to condition (15), which defines the asymptotic behavior of the pressure in an nonisothermal conducting gas film at finite Hartmann numbers. In [5] it is indicated that the carrying capacity of an isothermal bearing can be either increased or decreased by varying the direction of the H_0 vector and the value of the Hartmann number; undoubtedly, this thesis holds also for a nonisothermal bearing. However, the quantitative aspect of this effect was not examined. On the other hand, a practical example was computed in order to evaluate the asymptotic behavior of the pressure. The parameters chosen were: n = 3/4, ($\nu = 4/11$), $\chi = 2$, the eccentricity of a radial bearing $\eta = 0.3$, and a Hartmann number G = 2.785. As can be seen from Fig. 2 (which shows the pressure distribution curves at $\Lambda \to \infty$ for the given value of G and for G = 0), the difference in the behavior of these curves is fairly substantial. This indicates that the behavior of the flow characteristics in a lubricating film for $\chi \neq 1$ and $G \neq 0$ cannot be determined solely by introducing corrections to known solutions.

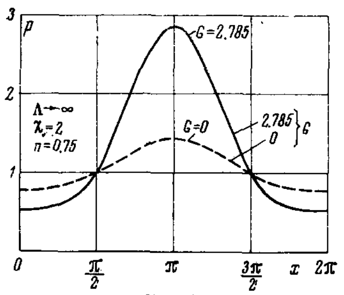

Fig. 2.

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